Wave Height Estimation in Stratified Gas-Liquid Flows

Andritsos and Hanratty (1978b) have shown that the increase in interfacial drag caused by waves in stratified gas-liquid flows is related to the wave steepness. Recent analyses of finite amplitude Kelvin-Helmholtz waves are used to develop a correlation for the ratio of the wave height to wavelength.

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Introduction

For low gas and liquid flow rates in a horizontal pipe, a stratified regime exists whereby liquid moves along the bottom of the pipe and the gas cocurrently above it. The prediction of interfacial drag is of key importance for calculating frictional pressure drop and liquid height (holdup) for this system. The most widely used method assumes that the drag at the interface, τ_i, is the same as for a flat surface.

It has been recognized that this assumption is incorrect because interfacial waves can have significant influence on τ_i. Andritsos and Hanratty (1987a,b) conducted experiments with horizontal pipelines of diameters 2.52 and 9.53 cm and with liquid viscosities of 1 to 80 mPa · s to determine τ_i. Their results indicate that two types of interfacial waves can exist: regular two-dimensional waves and large-amplitude irregular waves associated with a Kelvin-Helmholtz (K-H) instability. Increases in τ_i were found to be associated mainly with the large-amplitude K-H waves. The experiments of Andritsos and Hanratty for a very wide range of conditions showed that the ratio of interfacial friction factor, f_i, to the value for a smooth surface, f_G, is related to the ratio of the wave height to wavelength (wave steepness). Their original measurements, shown in Figure 1, demonstrate this dependence. This interesting result, however, cannot be used in developing design relations without having some reliable method to estimate wave properties from flow variables.

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The goal of the present work is to use recent results (Bontozoglou, 1988; Bontozoglou and Hanratty, 1988) from inviscid nonlinear wave theory to relate wave height to flow parameters. It is motivated by the observation by Andritsos and Hanratty (1987a) that the onset of large-amplitude waves is predicted by an inviscid K-H stability analysis.

In order to establish a criterion for wave heights, it is assumed that they correlate with the geometric limit (Saffman and Yuen, 1982; Bontozoglou and Hanratty, 1988). This limit occurs when the calculated wave slope becomes unphysical. Familiar examples of geometrically-limited free-surface waves are the Stokes gravity waves which develop a sharp corner at the crest and capillary waves for which the profile crosses itself at a critical height. Physically, the geometric limit is often associated with wave breaking and the onset of atomization.

A calculation of the geometrically-limited wave under conditions of an air flow has not been accomplished (although some comments have been made about the effect of current velocity on the shape of very steep waves by Bontozoglou and Hanratty, 1988). However, the exact steepness of this limiting wave could be of little practical use, since there is evidence (Banner and Phillips, 1974) that waves break well before they reach the calculated geometrically-limiting height.

The goal of this paper, however, is not to calculate exact wave heights but to discover what dimensionless groups are controlling. Numerical techniques developed by Saffman and Yuen (1982), and by Bontozoglou (1988) are used to calculate wave heights so as to determine the relative importance of the different dimensionless groups that are defined by inviscid theory. A criterion chosen to measure the closeness to breaking is the

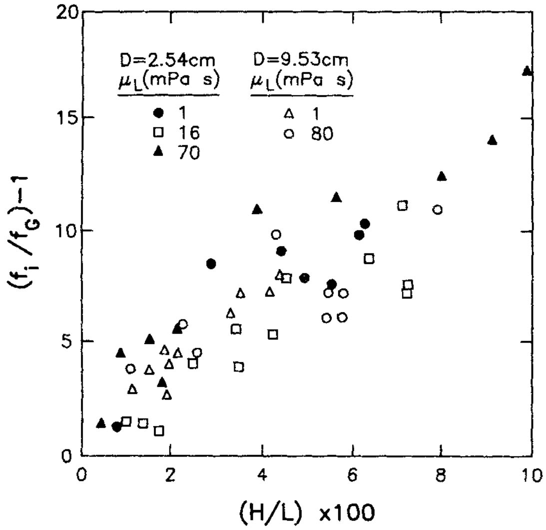

Figure 1. Effect of wave steepness on the interfacial friction factor.
From Andritsos and Hanratty (1987b).

ratio of the horizontal velocity of the fastest moving liquid particle on the interface, q, to the phase speed of the wave, $\epsilon = q/C$. There is evidence in the literature that this ratio is relevant to the geometric limit. Free-surface gravity waves, for example, are limited in height by a formation of a sharp peak at the crest when $\epsilon = 1$ (Stokes, 1847). The value of ϵ is found to increase montonically with an increase of wave height for waves in the presence of an air flow and $\epsilon = 1$ is associated either with a sharp peak at the crest or with an infinite slope elsewhere along the profile (Holyer, 1979; Bontozoglou and Hanratty, 1988). Because of numerical difficulties, the effect of flow parameters on the wave height corresponding to a given value of ϵ, rather than $\epsilon = 1$, was calculated. This is based on the assumption that waves with equal values of ϵ are equally close to the geometric limit.

A particularly interesting result from nonlinear wave theory is that the limiting wave steepness depends strongly on the thickness of the liquid on which the waves occur. Therefore, the wave height is not indicative, by itself, of how close to breaking the wave actually is. This is manifested by a calculated strong dependency of the wave height on the liquid height, in agreement with the empirical correlation of Andritsos and Hanratty (1987b).

Scaling by Inviscid Nonlinear Wave Theory

Progressive wave of permanent form at the interface between two fluids in relative motion is considered. The fluids are assumed inviscid and the flow, irrotational. The two streams have densities ρ_G, ρ_L, and uniform mean depths h_G, h_L, and move cocurrently with uniform velocities U_G, U_L. The flow is sketched in Figure 2. The interface is covered with two-dimensional, periodic waves of wavelength L (and wavenumber $k = 2\pi/L$) which propagate with phase speed of C in the direction of flow.

The assumption of waves with infinitesimally small amplitude (linearization) leads to the well-known Kelvin-Helmholtz instability, which may be interpreted as the nonexistence of steady, linear waves of a given wavelength when the relative velocity

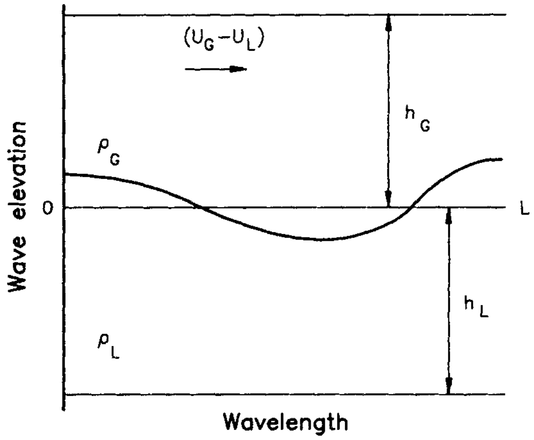

Figure 2. Flow system.

$(U_G - U_L)$ is larger than a critical U_{cl} (the subscript stands for critical linear). The value of U_{cl} is given by the expression

$$U_{cl}^2 = \frac{g}{k}\left(\frac{1+r}{r}\right)(1 - r + \kappa)(\tanh kh_G + r \tanh kh_L) \quad (1)$$

where r is the ratio of densities, ρ_G/ρ_L, and κ is the ratio of surface tension forces to the gravity forces, $k^2\sigma/\rho_L g$. For the experiments considered in this paper, $\tanh k\, h_G \simeq 1$ and $r \simeq 0$. Therefore, Eq. 1 may be simplified to

$$U_{cl}^2 = \frac{g}{k}\frac{(1+r)}{r}(1 - r + \kappa) \quad (2)$$

For finite amplitude waves with $L > 2.44$ cm, it is found that the critical current velocity, U_c, above which waves of permanent form cannot exist is larger than U_{cl}. Bontozoglou and Hanratty (1988) showed that U_c is an increasing function of wave height for all, but extremely thin films (or very long wavelengths). The wave steepness corresponding to U_c is called the dynamical limit, for which waves of permanent form with steepnesses less than this cannot exist. For $L > 2.44$, the dynamical limit is zero for $(U_G - U_L) \leq U_{cl}$ and is a finite increasing value for $(U_G - U_L) > U_{cl}$.

Because of the finding that interfacial drag increases dramatically for $(U_G - U_L) > U_{cl}$, it seemed natural to use U_{cl} as a characteristic velocity. Adopting the wavelength, L, as the characteristic length inviscid wave theory indicates that the geometrically limiting wave height is given by

$$\left(\frac{H}{L}\right)_{\lim} = f\left[\frac{\rho_G}{\rho_L}, \frac{k^2\sigma}{\rho_L g}, \frac{h_L}{L}, \frac{h_G}{L}, \frac{(U_G - U_L)}{U_{cl}}\right] \quad (3)$$

The first and second groups on the right side are the ratio of the gas density to the liquid density, r, and the ratio of surface tension to gravity forces, κ, defined after Eq. 1. Large values of h_L/L or h_G/L indicate that the height of the liquid or the gas space is not affecting the wave motion.

The assumption made in this paper is that the steepness of waves observed in gas-liquid flows scales the same as $(H/L)_{\lim}$, so that H/L is given by the functionality expressed in Eq. 3.

Some further simplifications of Eq. 3 can be made based on the data of Andritsos (1986). The wavelengths observed are typically larger than 4 cm, so additional effects of κ, apart from its influence on U_{cl}, are expected to be minor. (This has been verified by numerical studies of the type outlined in the following section.) In the stratified flow regime, h_G is typically an order of magnitude larger than h_L. The wavelengths are such that shallow gas effects are negligible and the influence of h_G/L can be ignored. (As a rule of thumb, the gas may be considered deep for $h_G > 0.3L$.) These assumptions are expected to hold even better for pipe diameters larger than the ones considered here. Under these simplifications, the dependence of the wave steepness H/L is given by

$$\left(\frac{H}{L}\right) = f\left[\frac{\rho_G}{\rho_L}, \frac{(U_G - U_L)}{U_{cl}}, \frac{h_L}{L}\right] \tag{4}$$

Numerical experiments will show that the function on the right side of Eq. 4 is separable, so that

$$\left(\frac{H}{L}\right) = \alpha\left(\frac{h_L}{L}\right) f\left(\frac{\rho_G}{\rho_L}, \frac{U_G - U_L}{U_{cl}}\right) \tag{5}$$

Further numerical experiments will also show that the effect of r is taken into account by using characteristic velocity, U_{cl}, defined by Eq. 2. Therefore, Eq. 5 can be further simplified to give

$$\frac{H}{L} = \alpha\left(\frac{h_L}{L}\right) f\left(\frac{U_G - U_L}{U_{cl}}\right). \tag{6}$$

Numerical Studies

Equation 5 was established by calculating the limiting wave height as a function of the liquid depth for both zero and high current velocities and for different density ratios. Geometrically-limited wave heights for free-surface gravity waves on liquids of arbitrary depth have been determined by Cokelet (1976). In Figure 3 the ratio, α, of the limiting height for free-surface waves ($r = 0$, $U = 0$) on a finite depth of liquid to the height on deep water ($\alpha = H/H_{deep}$), is plotted vs. the dimensionless liquid depth. The solid squares represent numerical results by Cokelet. These results for free-surface waves are also representative of gas-liquid waves at not very high pressures and zero current velocity.

The limiting wave heights for interfacial waves with current are not available in the current literature. However, a criterion has been adopted in the present work, which indicates how close a wave is to its limiting height. This criterion, discussed earlier, has the additional advantage that its definition does not depend on the liquid depth, so it unifies the results for all depths. Therefore, the parameter α, (defined in Eq. 5) has been calculated, not by comparing the actual limiting wave heights at different depths, but waves that are "equally close" to the geometrical limit (at constant values of ϵ).

Using a numerical method, developed by Saffman and Yuen (1982), to solve for progressive waves of permanent form, values of parameter α as a function of the dimensionless liquid depth were calculated. Results are shown in Figure 3 for ratios of fluid densities of $r = 0.001$ (gas-liquid at atmospheric conditions) and $r = 0.1$ (gas-liquid under high pressure). In both cases, the current velocity is 90% of the linear critical, U_{cl}. The results indicate that the relation between wave steepness and liquid depth is approximately independent of the current velocity and the ratio of fluid densities.

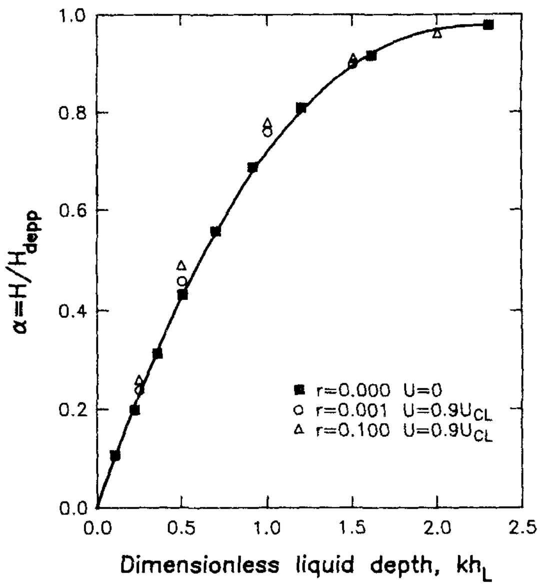

Figure 3. Reference steepness α as a function of the dimensionless liquid depth.

●, Free-surface waves and △, O, gas-liquid interfacial waves under high current velocities.

Calculations were also carried out to find the importance of $r = \rho_G/\rho_L$ in Eq. 5. Three density ratios were used, $r = 0.001$, 0.01, 0.1. Comparisons were made for large enough h_L/L that $\alpha(h_L/L) = 1.0$ at a constant value of $\epsilon = 0.22$. This specific value of ϵ was chosen so that the calculated H/L were in the range measured by Andritsos. The results, shown in Figure 4, support Eq. 6 by indicating that the effect of r is insignificant.

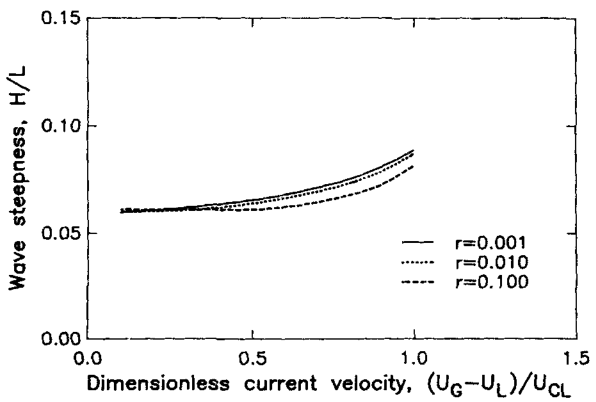

Figure 4. Wave steepness as a function of the dimensionless current velocity for various gas densities and a value of the steepness parameter, ϵ = 0.22.

Comparison with Measurements

One of the main purposes of this work was to compare the measurements of wave properties made by Andritsos (1986) with nonlinear wave theory. As indicated in Figure 1, these were carried out in horizontal pipelines, so that the geometry of the gas and liquid spaces did not correspond exactly to Figure 2. For the purpose of comparison, two definitions of h_L were explored. One of these was the distance from the bottom of the pipe to the mean liquid level; the second was the ratio of the cross-sectional area of the liquid to the width of the interface. Neither proved to be superior to the other in correlating the data, so the first definition was used because of its simplicity.

A common way of scaling wave heights for engineering practice is to assume that it is proportional to the liquid height. This notion has been applied reasonably successfully for the very thin films that exist in annular gas-liquid flows. However, it does not work for stratified flows. The data of Andritsos, for example, shows the values of H/h_L ranging from 0.09 to 1.7.

A plot of the Andritsos results in accordance with Eq. 6 is presented in Figure 5. The correlation is reasonable, in view of the fact that the reported wave properties are just statistical averages of a very complicated interfacial structure. Even more encouraging is that no measurable effect of pipe diameter or liquid viscosity is found. It should also be noted that almost half of the points are for atomizing waves, which clearly have reached their geometric limit. The results in Figure 5 are roughly fitted with the equation

$$\frac{H}{L} = 0.079\,\alpha\,\frac{(U_G - U_L)}{U_{cl}}, \tag{7}$$

where $\alpha\ (k\ h_L)$ is defined in Figure 3.

The use of Figure 1 and Eq. 7 suggests the following relation for the interfacial friction factor:

$$\frac{f_i}{f_G} - 1 = f\left[\alpha\left(\frac{U_G - U_L}{U_{cl}}\right)\right] \tag{8}$$

There is some similarity between this result and the empirically-derived equation of Andritsos and Hanratty (1987) for

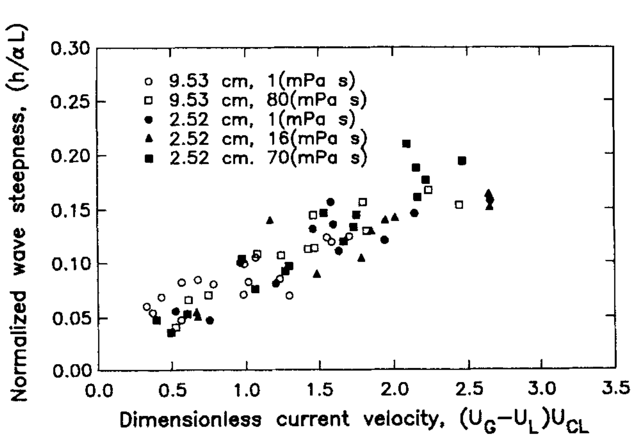

Figure 5. Normalized wave steepness vs. the dimensionless current velocity for all the data in the two pipe diameters.

$(U_G - U_L) > U_{cl}$:

$$\frac{f_i}{f_G} - 1 = 15\left(\frac{h_L}{D}\right)^{0.5}\left[\frac{(U_G - U_L)}{U_{cl}} - 1\right] \tag{9}$$

The strong effect of (kh_L) determined by the numerical calculations is represented here by the factor $(h_L/D)^{0.5}$.

One obstacle in the direct application of the present correlations to predict wave steepness, for design purposes, is the need for estimating the dominant wavelength L on the interface. Both the reference steepness α and the linear critical current U_{cl} depend on the wavelength. The prediction of the dominant wavelength from design parameters could require a consideration of the balance between energy transmitted to the waves of permanent form and the dissipation of energy in the liquid. This is essentially a different theoretical problem and is not considered in the present work.

However, if the wavelengths reported by Andritsos are examined, some empirical estimates can be obtained. For $(U_G - U_L) < U_{cl}$, the ratio of wavelength to liquid height, L/h_L, varies between 2.7 and 4.8 with 4.0 being a good average. A sudden increase of wavelength occurs at U close to U_{cl} and a sudden decrease occurs over a small range of gas velocities slightly larger than U_{cl}. (See Andritsos and Hanratty, 1987a, for a discussion of this behavior.) For still larger gas velocities and, in particular, for atomizing waves, the wavelength changes slowly with gas velocity. A range of 3–5 cm for the smaller diameter pipe and of 4–7 cm for the larger diameter pipe is found. These numbers can be used in the present correlations to derive a first estimate of the interfacial wave steepness, except in a short range of velocities close to U_{cl}. However, a better prediction of L is essential in making use of the full potential of the present work.

Acknowledgments

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Notation

C = wave velocity
f_i = friction factor = $\tau_i/(1/2)\rho\, U_G^2$
f_G = friction factor for a smooth interface
g = acceleration of gravity
H = wave height, the distance from the trough to the crest
h_G = average height of the gas layer
h_L = average height of the liquid layer
k = wavenumber = $2\pi/L$
L = wavelength
q = velocity of the fastest moving liquid particle at the interface
r = ratio of the densities = ρ_G/ρ_L
U_G = gas velocity
U_L = liquid velocity
U_{cl} = Kelvin-Helmholtz critical velocity predicted by linear theory

Greek letters

α = reference steepness defined by Eq. 5
κ = ratio of the surface tension to gravity forces = $k^2\sigma/\rho_L g$
ϵ = factor expressing the closeness to the dynamic limit = q/C
ρ_G = density of the gas
ρ_L = density of the liquid
σ = surface tension
τ_i = interfacial stress

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